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Research Article

An Adaptive Tracking Control of Fractional-Order Chaotic Systems with Uncertain System Parameter

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An adaptive tracking control scheme is presented for fractional-order chaotic systems with uncertain parameter. It is theoretically proved that this approach can make the uncertain parameter fractional-order chaotic system track any given reference signal and the uncertain system parameter is estimated through the adaptive tracking control process. Furthermore, the reference signal may belong to other integer-orders chaotic system or belong to different fractional-order chaotic system with different fractional orders. Two examples are presented to demonstrate the effectiveness of the proposed method.

1. Introduction

In nonlinear science, chaos synchronization is a hot topic, which has attracted much attention from scientists and engineers. In the past decades [1–13], various methodologies in control of chaotic system have been proposed, such as variable structure control approach, adaptive control approach, and time-delay feedback control. Up to date, the interest of the scientific community and engineering technological applications in controlling chaotic dynamics has increased after the development of the approach of "tracking," which can be used to follow an unstable fixed point or an unstable periodic orbit embedded or an arbitrary given reference signal in different dynamical regimes [3, 8–11]. In electronic and laser systems, the experiments on tracking the unstable periodic orbits and steady states were carried out [12, 13]. Some researchers design a controller based on the reference signal and make the output of the chaotic system follow the given reference signal successfully [10, 11].

However, many existed tracking control methods mainly focus on chaotic systems of integer orders [11, 14–17]. Up to now, there have been only a few papers on tracking control for fractional-order chaotic systems [18, 19]. In [18], tracking control of the fractional-order hyper-chaotic Lü system was reported, in which all the system parameters were exactly known. In [19], an adaptive tracking control of the fractional-order hyper-chaotic Lorenz system with unknown parameters was also reported. But, the tracking control methods in [18, 19] were unusual and only suited for specific fractional-order chaotic system, and the given reference signals do not belong to different fractional-order chaotic systems.

Inspired by the above discussion, in this paper, a more universal adaptive tracking control method for fractional-order chaotic system with uncertain system parameter, which is different from the previous works, is given based on the stability theory of fractional-order system. Furthermore, the reference signal may belong to other integer-orders chaotic system or belong to different fractional-order chaotic system with different fractional orders. To illustrate the effectiveness of the proposed scheme, we take 3D fractional-order Lorenz chaotic system with uncertain system parameter tracking the arbitrary given reference signal and a new 3D fractional-order chaotic system with uncertain system parameter tracking the 3D fractional-order Lorenz chaotic system, for example. Numerical simulations are performed to verify the effectiveness of proposed scheme.

The paper is organized as follows. In Section 2, an adaptive tracking control scheme is presented. In Section 3, two groups of examples are used to verify the effectiveness of the proposed scheme. The conclusion is finally drawn in Section 4.

2. Adaptive Tracking Control Scheme for Fractional-Order Chaotic Systems with Uncertain System Parameter

There are several definitions of fractional derivatives. In this paper, the Caputo-type fractional derivative will be used. The Caputo definition of the fractional derivative, which sometimes is called smooth fractional derivative, is described as

$$\frac{d^q f(t)}{dt^q} \equiv D^q f(t) = \frac{1}{\Gamma(m-q)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{q+1-m}} d\tau, \quad m-1 < q < m, \tag{2.1}$$

where m is the smallest integer larger than q, $f^{(m)}(t)$ is the m-order derivative in the usual sense, and $\Gamma(\cdot)$ stands for gamma function.

Now, we consider the fractional order system denoted as system (2.2) below

$$D^q \mathbf{x} = \mathbf{f}(\mathbf{x}, \alpha), \tag{2.2}$$

where $\mathbf{x} \in R^n$ and $\mathbf{f} : R^n \rightarrow R^n$ are differentiable functions. Parameter α is one of the system parameters.

When parameter α is unknown, a controller $\boldsymbol{\kappa}$ is added to the original system (2.2), we obtain

$$D^q \mathbf{x} = \mathbf{f}(\mathbf{x}, \tilde{\alpha}) + \boldsymbol{\kappa}, \tag{2.3}$$

where $\tilde{\alpha}$ is an unknown parameter to be estimated and $\boldsymbol{\kappa}$ is an $n \times 1$ real matrix to be designed.

Let $\mathbf{y} \in R^n$ be an arbitrary given reference signal. Our goal is to design the controller $\boldsymbol{\kappa}$ with an adaptive parameter such that the output signal $\mathbf{x}$ of system (2.3) follows the reference signal $\mathbf{y}$ ultimately and the uncertain parameter $\tilde{\alpha}$ could be identified. That is

$$\lim_{t \to +\infty} \|\mathbf{e}\| = \lim_{t \to +\infty} \|\mathbf{x} - \mathbf{y}\| = 0, \qquad \lim_{t \to +\infty} e_\alpha = \lim_{t \to +\infty} (\tilde{\alpha} - \alpha) = 0, \tag{2.4}$$

where $\| \cdot \|$ is the Euclidean norm and α is the "true" value of the "unknown" parameter $\tilde{\alpha}$.

Remark 2.1. The reference signal $\mathbf{y} \in R^n$ may belong to other integer-orders chaotic system or belong to different fractional-orders chaotic system with different fractional orders.

First, the controller $\boldsymbol{\kappa}$ is designed as follows:

$$\boldsymbol{\kappa} = \boldsymbol{\kappa}_1(\mathbf{y}) + \boldsymbol{\kappa}_2, \tag{2.5}$$

where $\boldsymbol{\kappa}_1(\mathbf{y})$ and $\boldsymbol{\kappa}_2$ are $n \times 1$ real matrix. $\boldsymbol{\kappa}_1(\mathbf{y})$ and $\boldsymbol{\kappa}_2$ are a compensation controller and a feedback controller, respectively. If the true value of the "unknown" parameter $\tilde{\alpha}$ is chosen as α, then the definition of the compensation controller $\boldsymbol{\kappa}_1(\mathbf{y})$ is

$$\boldsymbol{\kappa}_1(\mathbf{y}) = D^q \mathbf{y} - \mathbf{f}(\mathbf{y}, \alpha). \tag{2.6}$$

Remark 2.2. For defining the controllers (2.6) one has to know already the correct value of the parameter α, this point is the same as the master-slave (or drive-response) adaptive chaos synchronization with uncertain parameter. In master-slave (or drive-response) adaptive chaos synchronization with uncertain parameter, the "true" value of unknown parameter in slave system (or response system) is the correct value of parameter in master system (or drive system).

Let tracking errors $e_i = x_i - y_i$ $(i = 1, 2, \ldots, n)$ and parameter error $e_\alpha = \tilde{\alpha} - \alpha$. So, (2.3) can be changed as

$$D^q \mathbf{e} = \mathbf{f}(\mathbf{x}, \tilde{\alpha}) - \mathbf{f}(\mathbf{y}, \alpha) + \boldsymbol{\kappa}_2(\mathbf{x}, \mathbf{y}). \tag{2.7}$$

In generally, we can assume

$$\mathbf{f}(\mathbf{x}, \tilde{\alpha}) - \mathbf{f}(\mathbf{y}, \alpha) = \boldsymbol{\rho}_1(\mathbf{x}, \mathbf{y}, \alpha) \begin{pmatrix} \mathbf{e} \\ e_\alpha \end{pmatrix}, \tag{2.8}$$

where $\boldsymbol{\rho}_1(\mathbf{x}, \mathbf{y}, \alpha)$ is an $n \times (n+1)$ real matrix, $\mathbf{e} = (e_1 \ \ e_2 \ \ \cdots \ \ e_n)^{\mathbf{T}}$, and $\begin{pmatrix} \mathbf{e} \\ e_\alpha \end{pmatrix}$ is an $(n+1) \times 1$ real matrix.

Now, the feedback controller $\boldsymbol{\kappa}_2$ is chosen as,

$$\boldsymbol{\kappa}_2 = \boldsymbol{\rho}_2(\mathbf{x}, \mathbf{y}, \alpha) \begin{pmatrix} \mathbf{e} \\ e_\alpha \end{pmatrix}, \tag{2.9}$$

where $\boldsymbol{\rho}_2(\mathbf{x}, \mathbf{y}, \alpha)$ is an $n \times (n+1)$ real matrix to be designed later. According to (2.8) and (2.9), (2.3) or (2.7) can be rewritten as

$$D^q \mathbf{e} = \left[\boldsymbol{\rho}_1(\mathbf{x}, \mathbf{y}, \alpha) + \boldsymbol{\rho}_2(\mathbf{x}, \mathbf{y}, \alpha)\right] \begin{pmatrix} \mathbf{e} \\ e_\alpha \end{pmatrix}. \tag{2.10}$$

Second, the parameter update laws is chosen as

$$D^q \tilde{\alpha} = \alpha_{n+1,1} e_1 + \alpha_{n+1,2} e_2 + \cdots + \alpha_{n+1,n} e_n + \alpha_{n+1,n+1} e_\alpha = \boldsymbol{\alpha} \begin{pmatrix} \mathbf{e} \\ e_\alpha \end{pmatrix}, \tag{2.11}$$

where $\boldsymbol{\alpha}$ is an $1 \times (n+1)$ real matrix. Because the Caputo derivative of a constant is zero, so (2.11) can be rewritten as

$$D^q e_\alpha = D^q (\tilde{\alpha} - \alpha) = \boldsymbol{\alpha} \begin{pmatrix} \mathbf{e} \\ e_\alpha \end{pmatrix}. \tag{2.12}$$

According to (2.10) and (2.12), we can obtain

$$\begin{pmatrix} D^q \mathbf{e} \\ D^q e_\alpha \end{pmatrix} = \begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{x}, \mathbf{y}, \alpha) + \boldsymbol{\rho}_2(\mathbf{x}, \mathbf{y}, \alpha) \\ \boldsymbol{\alpha} \end{pmatrix} \begin{pmatrix} \mathbf{e} \\ e_\alpha \end{pmatrix}, \tag{2.13}$$

where $\begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{x},\mathbf{y},\alpha)+\boldsymbol{\rho}_2(\mathbf{x},\mathbf{y},\alpha) \\ \boldsymbol{\alpha} \end{pmatrix}$ is an $(n+1) \times (n+1)$ real matrix.

By (2.13), we know that the fractional-order system (2.3) tracking an arbitrary given reference signal $\mathbf{y}$ and the uncertain parameter $\tilde{\alpha}$ that could be identified are transformed into the following problem: choose a suitable matrix $\boldsymbol{\rho}_2(\mathbf{x}, \mathbf{y}, \alpha)$ and $\boldsymbol{\alpha}$ such that system (2.13) converges asymptotically to zero.

Theorem 2.3. *For an arbitrary given reference signal* $\mathbf{y}$, *the synchronization between fractional-order systems* (2.3) *and reference signal* $\mathbf{y}$ *will be obtained and the uncertain parameter* $\tilde{\alpha}$ *will be estimated if*

$$\mathbf{P} \begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{x}, \mathbf{y}, \alpha) + \boldsymbol{\rho}_2(\mathbf{x}, \mathbf{y}, \alpha) \\ \boldsymbol{\alpha} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{x}, \mathbf{y}, \alpha) + \boldsymbol{\rho}_2(\mathbf{x}, \mathbf{y}, \alpha) \\ \boldsymbol{\alpha} \end{pmatrix}^{\mathbf{H}} \mathbf{P} = -\mathbf{Q}, \tag{2.14}$$

where $\mathbf{P}$ *is a real symmetric positive definite matrix,* $\mathbf{Q}$ *is a real symmetric positive semidefinite matrix, and* $\mathbf{H}$ *stands for conjugate transpose of a matrix.*

Proof. Assume that λ is one of the eigenvalues of matrix $\begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{x},\mathbf{y},\alpha)+\boldsymbol{\rho}_2(\mathbf{x},\mathbf{y},\alpha) \\ \boldsymbol{\alpha} \end{pmatrix}$ and the corresponding nonzero eigenvector is β, that is,

$$\begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{x}, \mathbf{y}, \alpha) + \boldsymbol{\rho}_2(\mathbf{x}, \mathbf{y}, \alpha) \\ \boldsymbol{\alpha} \end{pmatrix} \beta = \lambda \beta. \tag{2.15}$$

Multiply the above equation left by $\beta^{\mathbf{H}}\mathbf{P}$, we derive that

$$\beta^{\mathbf{H}}\mathbf{P}\left(\begin{pmatrix}\boldsymbol{\rho}_1(\mathbf{x},\mathbf{y},\alpha)+\boldsymbol{\rho}_2(\mathbf{x},\mathbf{y},\alpha)\\ \boldsymbol{\alpha}\end{pmatrix}\beta\right)=\beta^{\mathbf{H}}\mathbf{P}(\lambda\beta). \tag{2.16}$$

Then, by a similar argument, we also can obtain that

$$\left(\beta^{\mathbf{H}}\begin{pmatrix}\boldsymbol{\rho}_1(\mathbf{x},\mathbf{y},\alpha)+\boldsymbol{\rho}_2(\mathbf{x},\mathbf{y},\alpha)\\ \boldsymbol{\alpha}\end{pmatrix}^{\mathbf{H}}\right)\mathbf{P}\beta=\left(\overline{\lambda}\beta^{\mathbf{H}}\right)\mathbf{P}\beta. \tag{2.17}$$

From (2.16) and (2.17), we can obtain

$$\lambda+\overline{\lambda}=\frac{\left\{\beta^{\mathbf{H}}\left[\mathbf{P}\begin{pmatrix}\rho_1(\mathrm{x,y},\alpha)+\rho_2(\mathrm{x,y},\alpha)\\ \boldsymbol{\alpha}\end{pmatrix}+\begin{pmatrix}\rho_1(\mathrm{x,y},\alpha)+\rho_2(\mathrm{x,y},\alpha)\\ \boldsymbol{\alpha}\end{pmatrix}^{\mathbf{H}}\mathbf{P}\right]\beta\right\}}{\beta^{\mathbf{H}}\mathbf{P}\beta}. \tag{2.18}$$

Since $\mathbf{P}\begin{pmatrix}\rho_1(\mathrm{x,y},\alpha)+\rho_2(\mathrm{x,y},\alpha)\\ \boldsymbol{\alpha}\end{pmatrix}+\begin{pmatrix}\rho_1(\mathrm{x,y},\alpha)+\rho_2(\mathrm{x,y},\alpha)\\ \boldsymbol{\alpha}\end{pmatrix}^{\mathbf{H}}\mathbf{P}=-\mathbf{Q}$ and $\mathbf{P}$, $\mathbf{Q}$ are real symmetric positive definite matrix and real symmetric positive semidefinite matrix, respectively, then

$$\beta^{\mathbf{H}}\mathbf{Q}\beta\geq 0,\qquad \beta^{\mathbf{H}}\mathbf{P}\beta>0, \tag{2.19}$$

so

$$\lambda+\overline{\lambda}=-\frac{\beta^{\mathbf{H}}\mathbf{Q}\beta}{\beta^{\mathbf{H}}\mathbf{P}\beta}\leq 0. \tag{2.20}$$

From (2.20), we have

$$\left|\arg\lambda\right|\geq\frac{\pi}{2}>\frac{q\pi}{2}. \tag{2.21}$$

According to the stability theory of fractional-order systems [20], the equilibrium point in (2.13) is asymptotically stable.

Therefore,

$$\lim_{t\to+\infty}e_i=\lim_{t\to+\infty}(x_i-y_i)=0\quad(i=1,2,\ldots,n),\qquad \lim_{t\to+\infty}e_\alpha=\lim_{t\to+\infty}(\tilde{\alpha}-\alpha)=0, \tag{2.22}$$

which indicates that the output signal $\mathbf{x}$ of fractional-order chaotic system (2.3) can track the reference signal $\mathbf{y}$ ultimately and the uncertain parameter $\tilde{\alpha}$ will be estimated. The proof is completed. □

3. Illustrative Example

To illustrate the effectiveness of the proposed scheme, two examples are considered and their numerical simulations are performed.

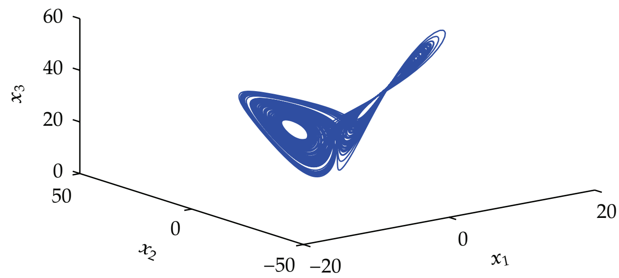


Figure 1: Chaotic attractors of the fractional-order Lorenz system for $q = 0.998$.

3.1. The 3D Fractional-Order Lorenz Chaotic System with Uncertain Parameter Tracking the Arbitrary Given Reference Signal

The 3D fractional-order Lorenz [21] system is given by

$$\begin{aligned}\frac{d^q x_1}{dt^q} &= \sigma(x_2 - x_1),\\ \frac{d^q x_2}{dt^q} &= \alpha x_1 - x_1 x_3 - x_2,\\ \frac{d^q x_3}{dt^q} &= x_1 x_2 - \beta x_3,\end{aligned} \tag{3.1}$$

where system parameters $(\sigma, \alpha, \beta) = (10, 28, 8/3)$. The fractional-order Lorenz system exhibits chaotic behavior for fractional-order $0.993 \leq q < 1$. The chaotic attractor for $q = 0.998$ is shown in Figure 1.

Let us assume that parameter α is unknown in the fractional-order Lorenz system. The fractional-order Lorenz system with uncertain parameter $\tilde{\alpha}$ is given by

$$\begin{aligned}\frac{d^q x_1}{dt^q} &= \sigma(x_2 - x_1),\\ \frac{d^q x_2}{dt^q} &= \tilde{\alpha} x_1 - x_1 x_3 - x_2,\\ \frac{d^q x_3}{dt^q} &= x_1 x_2 - \beta x_3.\end{aligned} \tag{3.2}$$

Now, we investigate the fractional-order Lorenz chaotic system with uncertain system parameter tracking the arbitrary given reference signal. We take reference signal $\mathbf{y} = (10 \sin t, 5 \cos t, 5)^{\mathbf{T}}$, for example. According to above mentioned, we can obtain

$$\rho_1(\mathbf{x}, \mathbf{y}, \alpha) = \begin{pmatrix} -10 & 10 & 0 & 0 \\ \alpha - x_3 & -1 & -10 \sin t & x_1 \\ x_2 & 10 \sin t & -\dfrac{8}{3} & 0 \end{pmatrix}. \tag{3.3}$$

Now, the parameter update laws and real matrix $\boldsymbol{\rho}_2(\mathbf{x},\mathbf{y},\alpha)$ are chosen as

$$D^q\tilde{\alpha} = -x_1e_2, \qquad \boldsymbol{\rho}_2(\mathbf{x},\mathbf{y},\alpha) = \begin{pmatrix} 0 & x_3-\alpha-10 & -x_2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.4}$$

Therefore,

$$\begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{x},\mathbf{y},\alpha)+\boldsymbol{\rho}_2(\mathbf{x},\mathbf{y},\alpha) \\ \boldsymbol{\alpha} \end{pmatrix} = \begin{pmatrix} -10 & x_3-\alpha & -x_2 & 0 \\ \alpha-x_3 & -1 & -10\sin t & x_1 \\ x_2 & 10\sin t & -\dfrac{8}{3} & 0 \\ 0 & -x_1 & 0 & 0 \end{pmatrix}. \tag{3.5}$$

Choosing real symmetric positive definite matrix $\mathbf{P} = \operatorname{diag}(1,1,1,1)$, we can yield

$$\mathbf{P}\begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{x},\mathbf{y},\alpha)+\boldsymbol{\rho}_2(\mathbf{x},\mathbf{y},\alpha) \\ \boldsymbol{\alpha} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{x},\mathbf{y},\alpha)+\boldsymbol{\rho}_2(\mathbf{x},\mathbf{y},\alpha) \\ \boldsymbol{\alpha} \end{pmatrix}^{\mathbf{H}}\mathbf{P} = \operatorname{diag}\left(-20,-2,-\frac{16}{3},0\right). \tag{3.6}$$

Choosing symmetric positive semidefinite matrix $\mathbf{Q} = \operatorname{diag}(20,2,16/3,0)$, we can obtain

$$\mathbf{P}\begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{x},\mathbf{y},\alpha)+\boldsymbol{\rho}_2(\mathbf{x},\mathbf{y},\alpha) \\ \boldsymbol{\alpha} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{x},\mathbf{y},\alpha)+\boldsymbol{\rho}_2(\mathbf{x},\mathbf{y},\alpha) \\ \boldsymbol{\alpha} \end{pmatrix}^{\mathbf{H}}\mathbf{P} = -\mathbf{Q}. \tag{3.7}$$

According to the above Theorem, the fractional-order Lorenz system (3.2) with uncertain parameters $\tilde{\alpha}$ can track the reference signal $\mathbf{y} = (10\sin t, 5\cos t, 5)^{\mathbf{T}}$ ultimately, and the uncertain parameters $\tilde{\alpha}$ will be estimated. The corresponding numerical result is shown in Figure 2, in which the initial conditions are $\mathbf{x} = (1,1,2)^{\mathbf{T}}$, $\tilde{\alpha}(0) = 20$ for system (3.2), and $\varepsilon = (\sum_{i=1}^{3} {e_i}^2)^{1/2}$, the "true" value of the "unknown" parameter is chosen as $\alpha = 28$, respectively.

3.2. A New 3D Fractional-Order Chaotic System with Uncertain System Parameter Tracking the 3D Fractional-Order Lorenz Chaotic System

A new 3D fractional-order chaotic system [22, 23] is reported by Sheu et al. It is described by

$$\begin{aligned} \frac{d^{q'}z_1}{dt^{q'}} &= \alpha z_1 - z_2z_3, \\ \frac{d^{q'}z_2}{dt^{q'}} &= -\beta z_2 + z_1z_3, \\ \frac{d^{q'}z_3}{dt^{q'}} &= \frac{z_1z_2}{3} - \gamma z_3, \end{aligned} \tag{3.8}$$

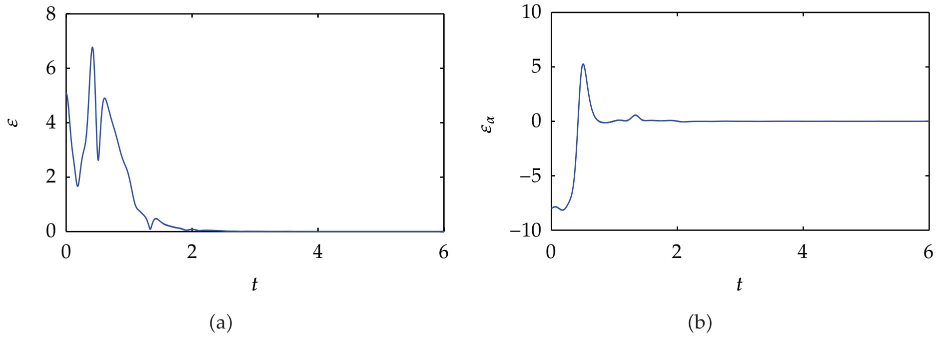


Figure 2: (a) shows tracking errors ε between the fractional-order Lorenz system (3.2) with uncertain parameter $\tilde{\alpha}$ and the reference signal $\mathbf{y} = (10\sin t, 5\cos t, 5)^{\mathrm{T}}$. (b) shows the parameter error ε_α between uncertain parameter $\tilde{\alpha}$ and the true value of the "unknown" parameter.

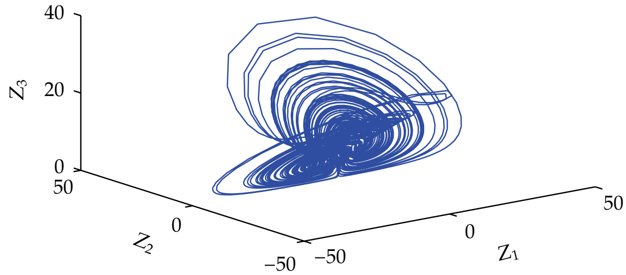


Figure 3: Chaotic attractor of a new fractional-order system (3.8) for $q' = 0.9$.

where system parameters $(\alpha, \beta, \gamma) = (5, 10, 3.8)$. The chaotic attractor for $q' = 0.9$ is shown in Figure 3.

Let us assume that parameter α is unknown in the fractional-order system (3.8). The fractional-order system with uncertain parameter $\tilde{\alpha}$ is described by

$$\begin{aligned}
\frac{d^{q'} z_1}{dt^{q'}} &= \tilde{\alpha} z_1 - z_2 z_3, \\
\frac{d^{q'} z_2}{dt^{q'}} &= -\beta z_2 + z_1 z_3, \\
\frac{d^{q'} z_3}{dt^{q'}} &= \frac{z_1 z_2}{3} - \gamma z_3.
\end{aligned} \tag{3.9}$$

Now, we investigate the fractional-order system (3.9) with uncertain system parameter $\tilde{\alpha}$ tracking the fractional-order Lorenz system (3.1). That is the reference signal belongs to the fractional-order Lorenz system (3.1), that is, $\mathbf{y} = (x_1, x_2, x_3)^{\mathrm{T}}$. So, the reference signal may belong to different fractional-order chaotic system with different fractional orders.

In this case, matrix $\boldsymbol{\rho}_1(\mathbf{x},\mathbf{y},\alpha)$ and $\boldsymbol{\rho}_2(\mathbf{x},\mathbf{y},\alpha)$ can be written as $\boldsymbol{\rho}_1(\mathbf{z},\mathbf{x},\alpha)$ and $\boldsymbol{\rho}_2(\mathbf{z},\mathbf{x},\alpha)$, respectively. According to above mentioned, we can obtain

$$\boldsymbol{\rho}_1(\mathbf{z},\mathbf{x},\alpha) = \begin{pmatrix} \alpha & -x_3 & -z_2 & z_1 \\ x_3 & -10 & z_1 & 0 \\ \dfrac{z_2}{3} & \dfrac{x_1}{3} & -3.8 & 0 \end{pmatrix}. \tag{3.10}$$

The parameter update laws and real matrix $\boldsymbol{\rho}_2(\mathbf{x},\tilde{\mathbf{x}},\alpha)$ are chosen as

$$D^q\tilde{\alpha} = -z_1e_1, \qquad \boldsymbol{\rho}_2(\mathbf{z},\mathbf{x},\alpha) = \begin{pmatrix} -\alpha-1 & 0 & \dfrac{2z_2}{3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -z_1-\dfrac{x_1}{3} & 0 & 0 \end{pmatrix}. \tag{3.11}$$

Therefore,

$$\begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{z},\mathbf{x},\alpha)+\boldsymbol{\rho}_2(\mathbf{z},\mathbf{x},\alpha) \\ \boldsymbol{\alpha} \end{pmatrix} = \begin{pmatrix} -1 & -x_3 & -\dfrac{z_2}{3} & z_1 \\ x_3 & -10 & z_1 & 0 \\ \dfrac{z_2}{3} & -z_1 & -3.8 & 0 \\ -z_1 & 0 & 0 & 0 \end{pmatrix}. \tag{3.12}$$

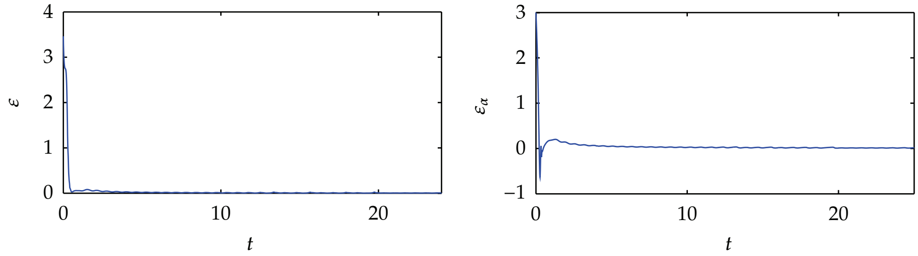
Choosing real symmetric positive definite matrix $\mathbf{P} = \operatorname{diag}(1,1,1,1)$, we can yield

$$\mathbf{P}\begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{z},\mathbf{x},\alpha)+\boldsymbol{\rho}_2(\mathbf{z},\mathbf{x},\alpha) \\ \boldsymbol{\alpha} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{z},\mathbf{x},\alpha)+\boldsymbol{\rho}_2(\mathbf{z},\mathbf{x},\alpha) \\ \boldsymbol{\alpha} \end{pmatrix}^{\mathbf{H}}\mathbf{P} = \operatorname{diag}(-2,-20,-7.6,0). \tag{3.13}$$

Choosing symmetric positive semidefinite matrix $\mathbf{Q} = \operatorname{diag}(2,20,7.6,0)$, we can obtain

$$\mathbf{P}\begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{z},\mathbf{x},\alpha)+\boldsymbol{\rho}_2(\mathbf{z},\mathbf{x},\alpha) \\ \boldsymbol{\alpha} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\rho}_1(\mathbf{z},\mathbf{x},\alpha)+\boldsymbol{\rho}_2(\mathbf{z},\mathbf{x},\alpha) \\ \boldsymbol{\alpha} \end{pmatrix}^{\mathbf{H}}\mathbf{P} = -\mathbf{Q}. \tag{3.14}$$

According to the above theorem, the fractional-order system (3.9) with uncertain parameter $\tilde{\alpha}$ can track the fractional-order Lorenz system (3.1) ultimately and the uncertain parameter $\tilde{\alpha}$ will be estimated. The corresponding numerical result is shown in Figure 4, in which the initial conditions are $\mathbf{x} = (1,1,2)^{\mathbf{T}}$ for fractional-order Lorenz system (3.1), and $z = (3,3,4)^{\mathbf{T}}$, $\tilde{\alpha}(0) = 8$ for fractional-order system (3.9), and $\varepsilon = (\sum_{i=1}^{3} e_i{}^2)^{1/2}$, the "true" value of the "unknown" parameter is chosen as $\alpha = 5$, respectively.



(a)

(b)

Figure 4: (a) shows tracking errors ε between the fractional-order system (3.9) with uncertain parameter $\tilde{\alpha}$ and the reference signal belongs to the fractional-order Lorenz system (3.1). (b) shows the parameter error ε_α between uncertain parameter $\tilde{\alpha}$ and the true value of the "unknown" parameter.

4. Conclusion

In this paper, a more universal adaptive tracking control scheme for fractional-order chaotic systems with uncertain parameter is addressed. Based on the stability theory of fractional-order system, an adaptive controller is designed and the parameter update law for estimating the unknown parameter of the systems is also gained. The 3D fractional-order Lorenz chaotic system with uncertain system parameter tracking the arbitrary given reference signal and a new 3D fractional-order chaotic system with uncertain system parameter tracking the 3D fractional-order Lorenz chaotic system are chosen to illustrate the proposed method. The numerical simulations demonstrate the validity and feasibility of the proposed method.

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